លំហាត់ទី ៣ សម្រាប់ថ្ងៃទី ១

គណនា: $S=1+x+x^2+...........+x^n$ និង $\sum=1+2.\frac{1}{3}+3.\frac{1}{3^2}+4.\frac{1}{3^3}+......+(n+1).\frac{1}{3^n}$

+ គណនា: $S=1+x+x^2+...........+x^n$

យើងមាន: $1-x^{n+1}=(1-x)(1+x+x^2+x^3+.........+x^n)$

នាំអោយ: $1+x+x^2+...........+x^n=\frac{1-x^{n+1}}{1-x}$

ដូច្នេះ $\boxed{1+x+x^2+...........+x^n=\frac{1-x^{n+1}}{1-x}}$

ដោយ $1+x+x^2+...........+x^n=\frac{1-x^{n+1}}{1-x}\Rightarrow 1+x+x^2+...........+x^{n+1}=\frac{1-x^{n+2}}{1-x}$

នាំអោយ: $1'+x'+(x^2)'+(x^3)'+.........+(x^{n+1})'=\left(\frac{1-x^{n+2}}{1-x}\right)'$

$$\Leftrightarrow 0+1+2x+3x^2+..........+(n+1)x^n=\frac{-(n+2)x^{n+1}(1-x)+(1-x^{n+2})}{(1-x)^2}$$

$$\Leftrightarrow 1+2x+3x^2+.......+(n+1)x^n=\frac{-(n+2)x^{n+1}+(n+2)x^{n+2}+1-x^{n+2}}{(1-x)^2}$$

$$\Leftrightarrow 1+2x+3x^2+.......+(n+1)x^n=\frac{(n+1)x^{n+2}-(n+2)x^{n+1}+1}{(1-x)^2}$$

ជំនួស x ដោយ តំលៃ $\left(\frac{1}{3}\right)$ យើងបាន:

$$1+2.\frac{1}{3}+3.\frac{1}{3^2}+4.\frac{1}{3^3}+.......+(n+1).\frac{1}{3^n}=\frac{(n+1)\frac{1}{3^{n+2}}-(n+2)\frac{1}{3^{n+1}}+1}{1-\frac{1}{3^2}}$$

$$=\frac{(n+1-3n-6+3^{n+2}).3^2}{3^{n+2}.4}=\frac{3^{n+2}-2n-5}{4.3^n}$$

ដូច្នេះ $\boxed{\sum=1+2.\frac{1}{3}++3.\frac{1}{3^2}+.........+(n+1).\frac{1}{3^n}=\frac{3^{n+2}-2n-5}{4.3^n}}$